DETERMINATION OF THE RELATIVE HEAT AND MASS EXCHANGE COEFFICIENTS AND THE CRITICAL SEPARATION PARAMETERS FOR A TURBULENT BOUNDARY LAYER UNDER INHOMOGENEOUS BLOWING IN CONDITIONS OF NONISOTHERMY

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Results of a determination of the relative limit laws of heat exchange and the separation parameters are presented for the general case of a density distribution over a boundary layer in subsonic flow. The results of computations on a digital computer are in good agreement with the proposed approximate formulas.

1. In subsonic multicomponent gas flow over a permeable surface (blowing, chemical reactions, etc.) under conditions of nonisothermy, the density distribution over the boundary layer with Prandtl and Lewis numbers Pr = Le = 1 is represented in general form by the expression

$$\frac{\rho_0}{\rho} = \frac{[\psi_1 + (1-\psi_1)\theta]\,[\psi_2 + (1-\psi_2)\theta]}{[\psi_3 + (1-\psi_3)\theta]}. \tag{1.1}$$

Here

$$\psi_1 = h_w/h_0; \quad \psi_2 = M_0/M_w; \quad \psi_3 = c_{pw}/c_{p0}.$$

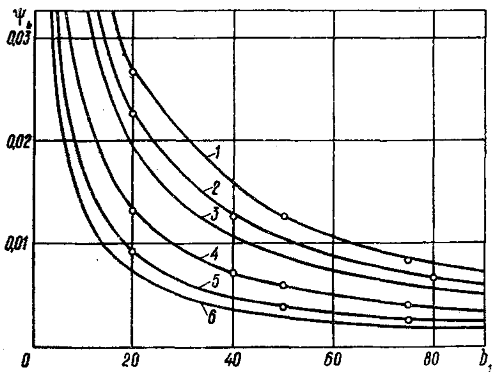

Fig. 1

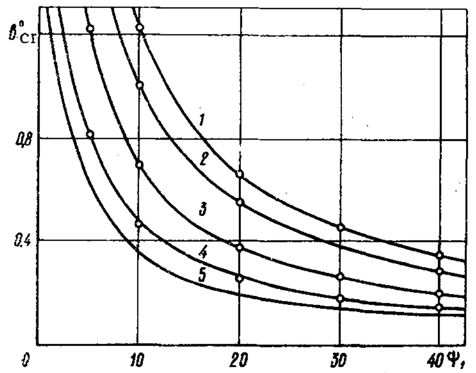

Fig. 2

Fig. 1. Relative coefficient of heat–mass-exchange for the values $\psi_1 = 20$ and $\psi_2 = 5$ and arbitrary ψ_3. 1-6) Machine computation; points) computation using (1.4) and (1.5): 1) $\psi_3 = 12.5$; 2) 10; 3) 8; 4) 5; 5) 3; 6) 2.

Fig. 2. Critical boundary layer separation parameters for $\psi_2 = 5$ and arbitrary values of ψ_3. 1-5) Machine computation; points) computation using (1.5): 1) $\psi_3 = 10$; 2) 8; 3) 5; 4) 3; 5) 2.

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The limiting relative heat and mass exchange law in the presence of a transverse stream of material is

$$\Psi_h = \left(\int_0^1 \frac{d\theta}{\sqrt{(1+b_1\theta)\rho_0/\rho}} \right)^2. \tag{1.2}$$

Here

$$\Psi_h = \mathrm{St}/\mathrm{St}_0; \quad b_1 = j_w/\gamma_0 u_0 \mathrm{St}.$$

The integral (1.2) has been evaluated on a digital computer taking account of (1.1) for arbitrary values of ψ_1, ψ_2, ψ_3 and b_1. Some results of the computations are presented in Fig. 1 (solid lines).

Data of a machine computation are presented in Fig. 2 for some values of ψ_1, ψ_2, ψ_3 for the boundary layer separation parameter defined by the expression [1]:

$$b_{\mathrm{cr}}^0 = \left(\int_0^1 \frac{d\theta}{\sqrt{(\rho_0/\rho)\,\theta}} \right)^2. \tag{1.3}$$

For practical computations, the following approximate formulas

$$\Psi_h = \left(\frac{2}{\sqrt{\psi_1}+1} \right)^2 \left(\frac{2}{\sqrt{\psi_2}+1} \right)^2 \left(\frac{\sqrt{\psi_3}+1}{2} \right)^2 \left(1 - \frac{b_1\Psi_h}{b_{\mathrm{cr}}^0} \right)^2, \tag{1.4}$$

$$b_{\mathrm{cr}}^0 = b_{\mathrm{cr}\,1} b_{\mathrm{cr}\,2}/b_{\mathrm{cr}\,3} \tag{1.5}$$

can be used respectively for the law of heat exchange and the separation parameters. Here $b_{\mathrm{cr}\,i}$ is found from the known relations [1]

$$b_{\mathrm{cr}\,i} = \frac{(\arccos(2-\psi_i)/\psi_i)^2}{\psi_i - 1} \quad \text{for} \quad \psi_i > 1, \tag{1.6}$$

$$b_{\mathrm{cr}\,i} = \left(\ln \frac{1+\sqrt{1-\psi_i}}{1-\sqrt{1-\psi_i}} \right)^2 \Big/ (1-\psi_i) \quad \text{for} \quad \psi_i < 1. \tag{1.7}$$

In Figs. 1 and 2 the points represent the results of calculating the relative heat-exchange coefficients and the parameters of forcing back the boundary layer by means of the proposed formulas (1.4) and (1.5). Good correspondence between the quantities being determined is observed upon a comparison with the results of a machine computation.

For the particular case of blowing a homogeneous gas $\psi_2 = \psi_3$, the relative heat-exchange coefficient is computed by means of the formulas [1]

$$\Psi_h = \frac{4}{(1-\psi_1)\,b_1} \left[\ln \frac{\sqrt{(1-\psi_1)(1+b_1)} + \sqrt{b_1}}{\sqrt{1-\psi_1} + \sqrt{\psi_1 b_1}} \right]^2 \quad \text{for} \quad \psi_1 < 1 \tag{1.8}$$

$$\Psi_h = \frac{4}{(\psi_1 - 1)\,b_1} \left[\operatorname{arctg} \sqrt{\frac{b_1}{(\psi_1-1)(1+b_1)}} - \operatorname{arctg} \sqrt{\frac{b_1\psi_1}{\psi_1 - 1}} \right]^2 \tag{1.9}$$

$$\text{for} \quad \psi_1 > 1.$$

It has been shown in [1] that (1.8) and (1.9) are approximated well by the formula

$$\Psi_h = \left(\frac{2}{\sqrt{\psi_1}+1} \right)^2 \left(1 - \frac{b_1\Psi_h}{b_{\mathrm{cr}}^0} \right)^2. \tag{1.10}$$

Here b_{cr}^0 is determined by means of (1.6) and (1.7).

Upon blowing an inhomogeneous gas under strongly nonisothermal conditions the correction to the parameters ψ_2 and ψ_3 can reach a significant value. For example, let us take the blowing of helium when the concentration of injected gas on the wall tends to one, in air ($T_0 = 3000$°K, $T_W = 290$°K). In this case $\psi_2 = 7.25$ and $\psi_3 = 1.24/0.327 = 3.79$ and the correction to the heat exchange coefficient (see Eq. (1.12)) is $[(\sqrt{7.25}+1)/(\sqrt{3.79}+1)]^2 = 1.57$ times.

Analogous estimates show that the correction to the quantity b_{cr}^0 reaches the same order.

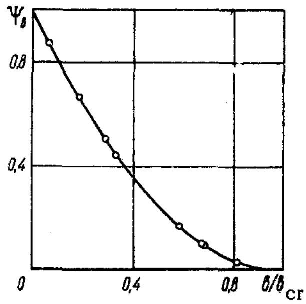

Fig. 3. Comparison between results of computing the relative heat–mass-exchange coefficients due to blowing by machine and by the approximate formula (1.12). Curve) computation by (1.12); points) machine computation for different values of ψ_1, ψ_2, ψ_3 and b_1.

Results of comparing the relative heat exchange coefficient during blowing, referred to isothermal conditions

$$\Psi_b = \Psi_h / \Psi_t, \tag{1.11}$$

with the approximating formula

$$\Psi_b = \left(1 - \frac{b_1 \Psi_h}{b^0_{cr}}\right)^2 \tag{1.12}$$

are presented in Fig. 3. Here

$$\Psi_t = \left(\frac{2}{\sqrt{\psi_1} + 1}\right)^2 \left(\frac{2}{\sqrt{\psi_2} + 1}\right)^2 \left(\frac{\sqrt{\psi_3} + 1}{2}\right)^2. \tag{1.13}$$

It is seen that the approximation (1.12) yields good correspondence with the results of a numerical machine solution.

2. The approximate expressions obtained for the relative law (1.4) of heat–mass-exchange Ψ_h and the boundary layer separation parameter b^0_{cr} in (1.5) can be extended to finite Reynolds number Re**. The law [1, 3] (for the range $200 < \mathrm{Re}^{**} < 10^4$)

$$\mathrm{St}_0 = \frac{0.0128}{\mathrm{Re}^{**0.25} \mathrm{Pr}^{0.75}} \left(\frac{\mu_w}{\mu_0}\right)^{0.25} \tag{2.1}$$

is hence taken as the standard value of the heat–mass-exchange coefficient.

The finiteness of the numbers Re** in determining the critical flowing parameters is taken into account in [1]:

$$b_{cr} = b^0_{cr}\left[1 + \frac{0.83}{\mathrm{Re}^{**0.14}}\right]. \tag{2.2}$$

In conclusion, it can be noted that the proposed approximate expressions (1.4) and (1.5) agree well with the results of the machine computation. The enthalpy h, the specific heat c_p, and the molecular weight M are determined for gas mixtures on the wall and in the stream core at the wall and main stream temperatures, respectively.

NOTATION

Re**	is the characteristic Reynolds number defined by the thickness of the energy loss;
Pr	is the Prandtl number;
Le	is the Lewis–Semenov number,
ρ	is the density of the gas mixture;
$\theta = (h - h_w)/(h_0 - h_w)$	is the dimensionless enthalpy;
h	is the enthalpy of the gas mixture;
c_p	is the specific heat;
M	is the molecular weight of a multicomponent mixture;
Ψ_h	is the relative coefficient of heat–mass-exchange taking account of the influence of the transverse flux of material, nonisothermy, etc.;
St	is the Stanton number under real conditions;

St_0	is the Stanton number for standard conditions;
b_1	is the wall permeability parameter referred to St;
b_{cr}	is the critical boundary-layer separation parameter referred to the Stanton number St_0 under standard condition;
Ψ_t	is the relative heat–mass-exchange coefficient taking account of the influence of the parameters ψ_1, ψ_2, ψ_3 (Eq. (1.12));
μ	is the dynamic viscosity coefficient.

Subscripts

w	is the wall;
0	is the main stream.

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